Date: November 16
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Algebraic Geometry

The qual started out with Yunqing asking me to state Riemann Roch. I stated it and defined geometric and arithmetic genus. Then someone (Yunqing?) asked me to say something about Riemann Roch applied to a curve of genus one. I stated the very ample criterion and showed that 3p gives an embedding of degree 3 into P^2, and (using the degree genus formula) that in fact every smooth degree 3 curve in P^2 is genus 1.

After that, Bernd asked me for the canonical embedding for genus 4 curves. I sketched the proof that the canonical bundle gives an embedding, and then wrote down the ideal sheaf sequence twisted by two to get that there was a quadric in the ideal, and similarly to get that there was a cubic, so that it was a complete intersection of a quadric and cubic in P^3.

Next, David asked what I could say about quadric surfaces in P^3. I'd been thinking a lot about surfaces in general, so I wrote down the adjunction to try to compute the genus of a general hypersurface in P^3. I then remarked as an aside that I just learned that geometric genus was really only defined as the sections of the canonical divisor for smooth schemes (!!!) and only defined for singular things if they are birational to a smooth thing, so that means there are schemes for which the geometric genus *isn't defined* because we don't know if there's a smooth model, and isn't that crazy? Everyone nodded at my fun fact.

Anyway, I wasn't really sure what David was getting at/ what kind of information he wanted to compute about this hypersurface. He asked me to define the Picard group and write down some examples I know. Then he asked me to write down a quadric hypersurface and say what its Picard group was. I finally realized that I hadn't mentally specialized to n=2 and that he wanted me to say that the quadric surface was isomorphic to P^1 x P^1. I felt a bit silly but said what the Picard group was. Then Bernd asked me if I knew what it was for a cubic hypersurface; I blurted out Z / 7Z and then corrected the mental typo to Z^7. (You can see this by representing it as a blowup of 6 points in P^2). David remarked that I was the second person today who made that mental typo and that there must be something in the air.

Finally, Yunqing or David (I forget which) asked me about the line bundle 2p on a genus 1 curve. I said it gave a 2:1 morphism to P^1. Then I was asked to compute the number of branch points, which I did using Riemann-Hurwitz. Emily then asked me if she gave me some points, if I could write down a curve with those as branch points. I tried doing a local model, which gave me a quartic, and claimed that I could glue the two local models together. Everyone looked kind of confused, and wanted a degree 3 curve. I said to assume one point was at infinity and there was an audible "yep ok" from about three people.

Representation Theory

Bernd told me to write the character table for S_4. I wrote down four rows and decomposed the regular representation to get the last one. David asked me for a more explicit description of what the representation corresponding to the partition (2,2) was. I gave a definition as the image of multiplication of C[G] by a Young symmetrizer, and said I thought there was a more explicit description as the kernel of a certain map, but had forgotten it.

Then he asked me for an elementary proof that I could find all representations as sub-representations of the regular one. I stated the strong version of Schur's lemma, i.e. that dim Hom(V, W) for V an irrep is the number of times V appears in W. I then mentioned as an interesting aside that I had recently learned this theorem does not hold over R! I gave an example of a representation of Z/4Z over R which did not satisfy Schur's lemma. David seemed kind of confused so I went through it more slowly. At the end Bernd seemed confused as to why I was doing this example and asked what the upshot was. I said, it's that we're working over the complex numbers. Everyone laughed.

Then I said "so we just need to find a map from V to C[G]"! And David said "great! Which one did you have in mind?" and I said "I'm still figuring that out!" I then messed around a bit and wrote down the intertwining condition for a map of representations. I didn't have anything concrete within a couple minutes, but David said he was satisfied, so we ended the rep theory portion. Then Yunqing answered the question by saying "take the image of C[G] acting on any vector v– there's not a canonical choice." David was confused about left vs right actions, and they talked for several minutes about how to resolve it. At some point during this discussion I left to get water, thus initiating a break. David started telling me about the Peter Weyl theorem when I got back.

Positive Geometry

Bernd asked me to define the amplituhedron, which I did. I gave some examples where k=1 and m=1. I got mostly clarifying questions about notation here from the other committee members, rather than "thinking" questions.

Bernd then asked me about positroid varieties. We spent a while on the matroid stratification– Emily wanted to know what the strata were like, to which I waved my hands and said "Mnev universality" and that they weren't isomorphic to affine space, and that their singularities could be arbitrarily complicated. However, the positroid varieties were normal and had rational singularities. David wanted to know how the matroid stratification related to the Schubert stratification, to which I actually had a nice answer: the Schubert stratification records jump sequences to the standard flag, whereas matroid cells record jump sequences relative to all permutations of the standard flag. So it is a finer stratification given by intersecting Schubert stratifications. Yunqing wanted an example of a stratum, and I wrote one down for the matroid {13,14,23,24}. The closure ended up just being P1 x P1.

I then had a slightly philosophical discussion with David where he asked basically why I was interested in these and I finally allowed myself to say some overarching (and not super precise) things about wanting to understand how positroid cells behaved under projection, and how this relates to their cohomology class in the Grassmannian.

Finally, David asked how “morally” information about the real points could possibly determine niceness (reasonable singularities) in the complex points. I told him that was a very philosophical question and I needed a moment to see if I could think of a satisfying answer. Bernd chimed in to say that, given a matroid stratum, the property of meeting the positive Grassmannian was related to a lot of nice combinatorics, and the niceness of the complex points was related to the combinatorics, without necessarily relying on the positivity. We ended there, about 45 minutes early.

All in all it was a pretty fun experience, and I definitely felt like my committee was trying to get me to talk about things I knew rather than to trip me up. I highly recommend putting your “fun” or very specialized topic last, if you have one, so that you can enjoy talking about it stress-free.